Program	Type of studies (cycle)	Third cycle
	Name of the program	Doctoral Studies in Mathematical and Computer Science

Course

Course title	**Selected areas of analytic number theory**

Course code	Semester	Course status	ECTS credits	Contact hours
PMAT 686	II	Elective	10	30

Aims and intended learning outcomes

The aim of the course is to consider generalizations of the prime geodesic theorem PGT (prime number theorem PNT) for various locally symmetric spaces. In general, we discuss: compact locally symmetric spaces of real rank 1, and, particularly: compact symmetric spaces attached to Lie group SL4, compact Riemann surfaces, and non-compact real hyperbolic manifolds with cusps. We are dealing with a summary of prevoius reearch from the part of analytic number theory that deals with PNT and its generalizations to spaces of higher dimensions and spaces of various compactness. In the literature, such generalizations are known as PGT or length spectrum theorems (of the corresponding operator). Generalizations of such generalizations by higher order counting functions are further known as weighted: prime number theorems, prime geodesic theorems, length spectrum. In the course we deal with such theorems. The preliminary material of each section is accompanied by auxiliary results necessary for that section. As usual, we omit proofs of auxiliary results. Alternative aspects of improving estimates in PGT variants, viewed through the prism of Gallagherisation, are presented at the end of the course.

Course content/topics

- Compact locally symmetric spaces of real rank 1. Cases: real, complex, quaternionic, octonionic. Casimir and Dirac operators. Principle of proportionality. Harmonic analysis on compact spaces. Admissible lifts. Trace formula elements. Theta functions. Zeta functions and the geodesic flow. Meromorphic continuation and singularities. Functional equations. Determinants and zeta functions. Duality principle. Differential forms. Order of Selberg's and Ruelle's zeta functions for compact even-dimensional locally symmetric spaces. PGT for compact spaces of rank 1. Asymptotic behavior of singularities. Logarithmic derivative of zeta functons. Consequences of PGT. Gallagherisation.
- Compact symmetric spaces of SL4. Euler characteristics. Euler-Poincare functions. Analysis of Ruelle zeta function. Selberg zeta function. Functional equation. Contribution of unit representation. PGT for SL4. Analytic properties of Ruelle zeta function. Counting functions. Approximate formulas. Refinement of the remainder. Length spectrum. The case of higher order counting function.
- Compact Riemann surfaces. Notation and normalization. Summing by the typical means. PGT and weighted PGT for compact Riemann surfaces.
- Non-compact real hyperbolic manifolds with cusps. Notation. Principal series representations and Hodge-Laplace operator. Scattering operator. Zeta functions of Selberg and Ruelle. PGT and weighted PGT for hyperbolic manifolds with cusps.
- Classification of some simple Lie algebras. Some groups of matrices and their Lie algebras. Octonions.

LITERATURE

[1] Dž. Gušić, *Generalizacije teorema o prostim brojevima*, PMF UNSA, Sarajevo, 2021.

[2] U. Bunke and M. Olbrich, *Selberg zeta and theta functions. A differential Operator Approach*, Akademie-Verlag, Berlin, 1995.

[3] M. Pavey, *Class numbers of orders in quartic fields*, Ph.D. dissertation, University of Tuebingen, 2006.

[4] D. Hejhal, *The Selberg trace formula for PSL(2,R) Vol. I, II*, Lecture notes in mathematics 548, 1001, Springer - Verlag 1973, 1983.

Grading

	Criterion	Points	Cut-off points
1.	Written assignment		
2.	Project		
3	Final exam		
	Total	100	55