| Program | Type of studies (cycle) | Third cycle |
|---|---|---|
| | Name of the program | Doctoral Studies in Mathematical and Computer Science |

**Course**

| Course title | **Algebraic Number Theory** | | | |
|---|---|---|---|---|
| Course code | Semester | Course status | ECTS credits | Contact hours |
| PMAT 645 | II | Elective | 10 | 30 |

| Aims and intended learning outcomes | Number theory is well known by challenging problems whose statements are accessible even to non-mathematicians, yet which resist sustained efforts to solve them over long periods. These efforts have shaped, and continue to shape, many mathematical disciplines. Major breakthroughs in recent decades and an unexpectedly broad range of applications have greatly increased research interest in number theory. Algebraic number theory applies methods of abstract algebra to number theoretic problems. The course aims to introduce students to selected current research areas in algebraic number theory. |
|---|---|

Course content/topics

- Algebraic numbers and algebraic integers
- Degrees; minimal polynomials; algebraic number fields
- Basis, norm, and trace of elements of number fields
- Discriminants; rings of integers
- Bases, norms, and discriminants of rings of integers; ideal norms
- Dedekind domains: ideal factorization in rings of integers
- Equivalent ideals; ideal classes; Dedekind's class number formula
- The ideal class group; unique factorization into prime ideals
- Decomposition of rational primes; ramification
- Units; Dirichlet's unit theorem
- Dedekind zeta functions and Hecke L-functions
- Elliptic curves over number fields
- Zeta functions of elliptic curves
- The Birch and Swinnerton-Dyer conjecture

**LITERATURE**


[1] N. Stewart and D. O. Tall, Algebraic Number Theory, 2nd ed., Chapman and Hall/CRC Press, 1987.

[2] H. P. F. Swinnerton-Dyer, A Brief Guide to Algebraic Number Theory, London Mathematical Society Student Texts 50, Cambridge University Press, Cambridge, 2001, x + 146 pp.

[3] J. Neukirch, Algebraic Number Theory, Grundlehren der Mathematischen Wissenschaften 322, Springer-Verlag, Berlin, 1999.

[4] K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, corrected 2nd printing, Graduate Texts 84, Springer, 1993.

[5] W. Narkiewicz, Elementary and Analytic Theory of Algebraic Numbers, 1990.


**Grading**

| | Criterion | Points | Cut-off points |
|---|---|---|---|
| 1. | Homework | 20 | |
| 2. | Project | 50 | |
| 3 | Final exam | 30 | |
| | Total | 100 | 55 |