Program	Type of studies (cycle)	Third cycle
	Name of the program	Doctoral Studies in Mathematical and Computer Science

Course

Course title	**Theory of L-functions: analytical and numerical approaches**			
Course code	Semester	Course status	ECTS credits	Contact hours
PMAT 606	II	Elective	10	30

Aims and intended learning outcomes	The course introduces contemporary *L*-function theory through the Selberg class and its generalizations, analytical properties of *L*-functions, the distribution of their zeros, and generalizations of the Riemann hypothesis. It focuses on functional equations, Euler products, zero-free regions, spectral and probabilistic methods, and numerical algorithms for computing *L*-functions and their zeros. Computational approaches using high-precision arithmetic, interval methods, and numerical software libraries are also examined. Upon completion, students will be able to analyze major classes of *L*-functions and their analytical properties, understand contemporary formulations and generalizations of the Riemann hypothesis, use asymptotic, spectral, and probabilistic methods, and implement numerical algorithms for evaluating *L*-functions and locating their zeros in analytic number theory research.

Course content/topics

- Dirichlet series, Euler products, and analytic continuation of *L*-functions
- Functional equations, gamma factors, and invariants of *L*-functions
- The Selberg class and its axiomatic structure
- Generalizations and modifications of the Selberg class
- Analytical properties, zero distributions, and zero-free regions
- Generalizations of the Riemann hypothesis in the Selberg class
- Explicit formulas and asymptotic methods in *L*-function theory
- Spectral, probabilistic, and random matrix methods for studying zeros
- Numerical methods for computing L-functions and their zeros
- High-precision algorithms, interval arithmetic, and the Arb library
- Computational experiments and contemporary problems in *L*-function theory

LITERATURE

[1] Henryk Iwaniec and Emmanuel Kowalski, Analytic Number Theory, American Mathematical Society, 2004.

[2] Hugh L. Montgomery and Robert C. Vaughan, Multiplicative Number Theory I: Classical Theory, Cambridge University Press, 2006.

[3] Jerzy Kaczorowski and Alberto Perelli, On the Structure of the Selberg Class, I–VI.

[4] Richard Crandall and Carl Pomerance, Prime Numbers: A Computational Perspective, 2nd ed., Springer, 2005.

[5] Fredrik Johansson, documentation and numerical methods for the Arb library.

Grading

	Criterion	Points	Cut-off points
1.	Homework		
2.	Project	50	
3	Final exam	50	
	Total	100	55