Program	Type of studies (cycle)	Third cycle – doctoral studies
	Name of the program	Doctoral Studies in Mathematical and Computer Science

Course

Course title	**Chaos Theory**			
Course code	Semester	Course status	ECTS credits	Contact hours
AMAT 676	II	elective	10	30

Aims and intended learning outcomes	The course aims to develop a theoretical understanding of complex dynamics in discrete and continuous dynamical systems, emphasizing sensitivity to initial conditions, topological chaos, symbolic dynamics, fractal structures, strange attractors, and mechanisms leading to chaotic behavior. The course covers methods for analyzing chaotic maps and flows, including one-dimensional maps, the logistic map, shift dynamics, Sharkovsky's theorem, Li–Yorke and Devaney chaos, topological transitivity, entropy, Lyapunov exponents, fractal dimensions, and classic examples of chaotic systems. Particular emphasis is placed on the relationship between local instability, the global structure of phase space, and long-term unpredictable behavior.

Course content/topics

- **Basic concepts of chaotic dynamics:** deterministic systems, long-term behavior of orbits, sensitivity to initial conditions, unpredictability, topological transitivity, density of periodic orbits, and different definitions of chaos.
- **One-dimensional maps and the transition to chaos:** interval maps, logistic map, tent map, unimodal maps, period-doubling cascade, bifurcation diagrams, and Feigenbaum constants.
- **Sharkovsky's theorem and Li–Yorke chaos:** periodic points of interval maps, Sharkovsky ordering, existence of period three, the Li–Yorke concept of chaos, scrambled sets, and implications for complex dynamics.
- **Symbolic dynamics:** shift spaces, itineraries, coding of orbits, subshifts of finite type, conjugacy and semiconjugacy, Markov partitions, and applications of symbolic dynamics to the analysis of chaos.
- **Topological chaos:** Devaney chaos, topological transitivity, topological mixing, sensitivity to initial conditions, density of periodic points, and topological equivalence of chaotic systems.
- **Topological entropy and measures of complexity:** topological entropy, growth in the number of periodic orbits, entropy of symbolic systems, and basic relationships among entropy, instability, and dynamical complexity.
- **Fractals and fractal dimensions:** self-similarity, the Cantor set, Hausdorff, box-counting, and correlation dimensions, fractal geometry of attractors, and the relationship between fractal structure and dynamics.
- **Lyapunov exponents and numerical diagnostics of chaos:** exponential separation of orbits, Lyapunov exponents for maps and flows, numerical computation of exponents, Poincaré sections, and time-series analysis.
- **Chaos in multidimensional maps:** the Hénon map, Smale horseshoe, homoclinic and heteroclinic structures, stretching and folding of phase space, and stable and unstable manifolds.
- **Chaos in continuous systems:** Lorenz, Rössler, and Chua systems, strange attractors, Poincaré maps, homoclinic mechanisms, and the transition from regular to chaotic behavior.
- **Numerical and experimental analysis of chaos:** bifurcation diagrams, attractor reconstruction, phase portraits, time series, numerical artifacts, limitations of simulations, and interpretation of chaotic dynamics.

LITERATURE

[1] Devaney, R. L. An Introduction to Chaotic Dynamical Systems. 3rd ed., Chapman & Hall/CRC, 2022.
[2] Alligood, K. T., Sauer, T. D., Yorke, J. A. Chaos: An Introduction to Dynamical Systems. Springer, 1996.
[3] Strogatz, S. H. Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering. 2nd ed., CRC Press, 2015.
[4] Wiggins, S. Introduction to Applied Nonlinear Dynamical Systems and Chaos. 2nd ed., Springer, 2003.
[5] Elaydi, S. N. Discrete Chaos: With Applications in Science and Engineering. 2nd ed., Chapman & Hall/CRC, 2007.
[6] Robinson, C. Dynamical Systems: Stability, Symbolic Dynamics, and Chaos. 2nd ed., CRC Press, 1999.
[7] Ott, E. Chaos in Dynamical Systems. 2nd ed., Cambridge University Press, 2002.

Grading

	Criterion	Points	Cut-off points
1.	Tests during the semester	50	27
2.	Final exam	50	28
	Total	100	55