Program	Type of studies (cycle)	Third cycle – doctoral studies
	Name of the program	Doctoral Studies in Mathematical and Computer Science

Course

Course title	**Continuous Dynamical Systems**			
Course code	Semester	Course status	ECTS credits	Contact hours
AMAT 655	II	elective	10	30

Aims and intended learning outcomes	The course aims to develop the theoretical foundations for analyzing flows generated by ordinary differential equations and their long-term behavior. Particular emphasis is placed on vector fields, linear and nonlinear systems, stability of equilibria, phase portraits, periodic solutions, the Poincaré map, limit sets, invariant manifolds, normal forms, and local bifurcations. The course develops skills for the qualitative analysis of continuous systems, including linearization, Lyapunov methods, Floquet theory, the Poincaré–Bendixson theorem, center manifold theory, and bifurcations of equilibria. It provides a theoretical foundation for further study of nonlinear dynamics, mathematical modeling, and chaos theory, without a detailed treatment of chaotic dynamics.

Course content/topics

- **Basic concepts of ordinary differential equations and continuous dynamical systems:** autonomous systems, existence and uniqueness of solutions, dependence on initial conditions and parameters, flows, orbits, phase space, and invariant sets.
- **Vector fields, linear systems, and phase portraits:** linear autonomous systems, matrix exponential, equilibria, classification of equilibria, phase portraits, stability, and asymptotic behavior of solutions.
- **Linearization and local stability:** Jacobian matrix, hyperbolic and nonhyperbolic equilibria, local stability, topological equivalence, and the Hartman–Grobman theorem.
- **Lyapunov methods:** Lyapunov functions, local and global stability, asymptotic stability, LaSalle's invariance principle, and basins of attraction.
- **Periodic solutions and Floquet theory:** periodic orbits, Floquet multipliers, Floquet theorem, matrix logarithm, and stability of periodic solutions.
- **Poincaré map:** Poincaré sections, construction of the Poincaré map, stability of periodic orbits, and local analysis.
- **Limit sets and planar dynamics:** alpha- and omega-limit sets, positive and negative invariance, limit cycles, phase plane, and the Poincaré–Bendixson theorem.
- **Normal forms and resonances:** reduction of systems near an equilibrium, resonant and nonresonant cases, Poincaré theorem, and local qualitative analysis.
- **Center, stable, and unstable manifolds:** invariant manifold theorems, reduction of dynamics to the center manifold, and approximate computation of the center manifold.
- **Local bifurcations of equilibria:** bifurcations with a zero eigenvalue, saddle-node, transcritical, pitchfork, and Hopf bifurcations, basic normal forms, and changes in stability.
- **Codimension-two bifurcations:** degenerate local bifurcations, cusp, Bogdanov–Takens, and Bautin bifurcations, two-parameter analysis, and bifurcation curves.
- **Attractors and more complex dynamics at an introductory level:** the concept of an attractor, stable and unstable sets, basins of attraction, and an introductory connection with complex dynamics, without a detailed treatment of chaos.

LITERATURE

[1] Hirsch, M. W., Smale, S., Devaney, R. L. Differential Equations, Dynamical Systems, and an Introduction to Chaos. 3rd ed., Academic Press, 2012.
[2] Wiggins, S. Introduction to Applied Nonlinear Dynamical Systems and Chaos. 2nd ed., Springer, 2003.
[3] Guckenheimer, J., Holmes, P. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer, 1983.
[4] Hale, J. K., Koçak, H., Dynamics and Bifurcations. Springer, 1991.
[5] Kuznetsov, Y. A. Elements of Applied Bifurcation Theory. 4th ed., Springer, 2023.

Grading

	Criterion	Points	Cut-off points
1.	Tests during the semester	50	27
2.	Final exam	50	28
	Total	100	55