Program	Type of studies (cycle)	Third cycle – doctoral studies
	Name of the program	Doctoral Studies in Mathematical and Computer Science

Course

Course title	**Discrete Dynamical Systems**			
Course code	Semester	Course status	ECTS credits	Contact hours
AMAT 616	II	elective	10	30

Aims and intended learning outcomes	The course aims to develop the theoretical foundations for analyzing iterative maps and difference equations. Particular emphasis is placed on the long-term behavior of orbits, stability of fixed and periodic points, local and global attractivity, invariant sets, Lyapunov methods, monotone maps, and local bifurcations of discrete systems. The course provides a foundation for further study of nonlinear dynamics and chaos theory, without a detailed treatment of symbolic, topological, and fractal theories of chaotic systems. It also provides the mathematical background for subsequent analysis of discrete models in applied mathematics and mathematical biology.

Course content/topics

The course covers the theoretical foundations of discrete dynamical systems, with an emphasis on stability, long-term orbit behavior, local analysis, and bifurcations of iterative maps.

- **Basic concepts of discrete dynamical systems:** iterative maps, orbits, fixed and periodic points, invariant sets, omega-limit sets, attractors, and basins of attraction.
- **Linear discrete systems:** scalar and multidimensional linear systems, matrix iterations, spectral radius, Jordan form, stability of the zero solution, and asymptotic behavior of orbits.
- **Local stability of fixed points:** linearization of maps, Jacobian matrix, eigenvalues, hyperbolic and nonhyperbolic fixed points, local asymptotic stability, and instability.
- **Stability of periodic orbits:** iterates of maps, periodic cycles, multipliers of periodic orbits, stable and unstable cycles, and local analysis of higher iterates.
- **Lyapunov methods and global attractivity:** discrete Lyapunov functions, local and global stability, absorbing sets, invariant regions, basins of attraction, and convergence of orbits.
- **One-dimensional discrete dynamics:** monotone and unimodal maps, graphical analysis of iterations, cobweb diagrams, fixed and periodic points, basic stability criteria, and the transition to more complex behavior.
- **Local and codimension-two bifurcations:** saddle-node, transcritical, pitchfork, period-doubling, and Neimark–Sacker bifurcations, basic normal forms, bifurcation diagrams, two-parameter analysis, degenerate cases, cusp bifurcation, and resonant cases.
- **Positive and monotone discrete systems:** positive maps, cones, partial order, monotone dynamics, cooperative and competitive maps, invariant regions, and preservation of positivity.
- **Multidimensional discrete dynamics:** local dynamics in the plane and higher dimensions, invertible and noninvertible maps, stable and unstable directions, and local stable and unstable manifolds.
- **Numerical analysis of discrete systems:** computation of orbits, fixed and periodic points, stability and bifurcation diagrams, estimation of basins of attraction, introductory Lyapunov exponents, and numerical artifacts.

LITERATURE

[1] Elaydi, S. N. An Introduction to Difference Equations. 3rd ed., Springer, 2005.
[2] Kulenović, M. R. S., Merino, O. Discrete Dynamical Systems and Difference Equations with Mathematica. Chapman & Hall/CRC, 2002.
[3] Wiggins, S. Introduction to Applied Nonlinear Dynamical Systems and Chaos. 2nd ed., Springer, 2003.
[4] Guckenheimer, J., Holmes, P. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer, 1983.
[5] Robinson, C. An Introduction to Dynamical Systems: Continuous and Discrete. American Mathematical Society, 2012.
[6] Kuznetsov, Y. A. Elements of Applied Bifurcation Theory. 4th ed., Springer, 2023.
[7] Bacciotti, A. Discrete Dynamics: Basic Theory and Examples. Springer, 2022.

Grading

	Criterion	Points	Cut-off points
1.	Tests during the semester	50	27
2.	Final exam	50	28
	Total	100	55