Study program	Type of study (cycle)	Third cycle of studies – doctoral studies
	Name of study program	Doctoral Study in Mathematical and Computer Sciences

COURSE

Course title	**Advanced Methods of Applied Mathematics**			
Course code	Semester	Course status	ECTS credits	Contact hours
AMAT 601	I	Elective	10	30

Course aims and learning outcomes	The course aims to deepen the mathematical methods that provide the theoretical basis for analyzing contemporary models in applied mathematics, particularly in dynamical systems, nonlinear analysis, and mathematical modeling. The course covers selected tools from spectral and functional analysis, fixed-point theory, nonlinear analysis, geometric methods, approximation methods, numerical methods, and basic stochastic methods. It places particular emphasis on convergence, stability, positivity, invariance, approximation of solutions, and the structure of phase spaces. The course provides theoretical preparation for a deeper study of discrete and continuous dynamical systems, differential and difference equations, operator methods, and mathematical models in the natural, engineering, and biomedical sciences.

Course content

The course will cover selected topics from the following areas, with the choice and depth of treatment depending on the study program's objectives and the candidate's research orientation.

- **Spectral analysis and matrix methods:** spectral radius, matrix norms, diagonalization, Jordan form, positive and nonnegative matrices, Perron-Frobenius theory, M-matrices, Metzler matrices, and stability of linear systems.
- **Normed, Banach, and Hilbert spaces:** completeness, compactness, compact operators, orthogonality, projections, self-adjoint and positive operators, spectral results, and basic operator methods.
- **Fixed-point theory and nonlinear analysis:** contractive, compact, and monotone mappings, Banach and Schauder fixed-point theorems, Fréchet and Gateaux differentiability, implicit and inverse function theorems, nonlinear operator equations, and Newton methods.
- **Manifolds and geometric methods:** smooth manifolds, local charts, tangent spaces, differentials of mappings, vector fields, local flows, and geometric interpretation of phase spaces and constraints in models.
- **Ordered spaces and positive operators:** cones, partial order, monotone mappings, positive linear and nonlinear operators, comparison methods, invariant regions, and preservation of positivity.
- **Fourier, approximation, and variational methods:** Fourier series and transforms, orthogonal expansions, projection methods, the Galerkin method, function approximation, weak convergence, and basic variational principles.
- **Operator and evolution methods:** densely defined and closed operators, fundamentals of operator semigroups, diffusion operators, weak formulations of problems, and basic concepts of existence, regularity, and stability of solutions.
- **Stochastic methods in applied mathematics:** random variables, basic distributions, Markov chains, transition matrices, stationary distributions, Poisson processes, branching processes, and fundamentals of stochastic models.
- **Numerical and computational methods:** conditioning of problems, stability of numerical procedures, iterative methods for linear and nonlinear systems, discretization of differential problems, approximation of solutions, and assessment of the reliability of numerical results.

LITERATURE

[1] Horn, R. A., Johnson, C. R. Matrix Analysis. 2nd ed., Cambridge University Press, 2012.
[2] Kreyszig, E. Introductory Functional Analysis with Applications. Wiley Classics Library, 1991.
[3] Berman, A., Plemmons, R. J. Nonnegative Matrices in the Mathematical Sciences. SIAM, 1994.
[4] Lee, J. M. Introduction to Smooth Manifolds. 2nd ed., Springer, 2013.
[5] Quarteroni, A., Sacco, R., Saleri, F. Numerical Mathematics. 2nd ed., Springer, 2007.
[6] Zeidler, E. Nonlinear Functional Analysis and its Applications I: Fixed-Point Theorems. Springer, 1986.
[7] Grimmett, G., Stirzaker, D. Probability and Random Processes. 4th ed., Oxford University Press, 2020.

ASSESSMENT AND GRADING

	Criterion	Points	Min.
1.	Tests during the semester	50	27
3	Final exam	50	28
	T o t a l	100	55