Program	Type of studies (cycle)	Third cycle
	Name of the program	SEE Doctoral Studies in Mathematical Science

Course

Course title	**Spectral theory of automorphic forms**

Course code	Semester	Course status	ECTS credits	Contact hours
	II		10	30

Teaching staff	Teacher	Prof. dr. Lejla Smajlović
	Other staff	

Course goals

The main goal of the course is to introduce basic aspects of spectral theory of automorphic forms on Fuchsian groups. To be precise, the goal is to describe spectral expansion of the space of cusp forms and the space of incomplete Eisenstein series and then to derive the Selberg trace formula on non-compact Riemannian surfaces of finite volume. Then, using the trace formula we will introduce the Selberg zeta function and describe its basic properties. As an application of the methods introduced, we will derive the Weyl law for the distribution of eigenvalues of the Laplacian on non-compact Riemannian surfaces of finite volume.

Course content/topics

- Harmonic analysis on hyperbolic plane: hyperbolic coordinates, classification of isometries, eigenfunctions of the Laplace operator, invariant integral operators, Selberg/Harish-Chandra transform.
- Fuchsian groups: definition of a Fuchsian group and its fundamental domain, classification of elements of Fuchsian groups, classification of Fuchsian groups, some special arithmetic Fuchsian groups.
- Automorphic forms: definition, definition of a cusp form and Eisenstein series, Kloosterman sums and Fourier expansion of Eisenstein series.
- Green's function on the upper half-plane and spectral expansion of the space of cusp forms (as Δ-invariant subspaces).
- Automorpric Green function and analytic continuation of Eisenstein series.
- Functional equation, poles and residues of Eisenstein series.
- Spectral expansion of the space of incomplete Eisenstein series (as Δ- invariant subspaces).
- The Selberg trace formula.
- The Selberg zeta function, its basic properties and functional equation.
- The Weyl law.

LITERATURE

[1] H. Iwaniec, *Spectral Methods of Automorphic Forms*, Graduate Studies in Mathematics, Vol. 53, American Mathematical Society, 2002.

[2] H. Iwaniec, E. Kowalski, *Analytic Number Theory*, AMS Colloquium Publications, Vol. 53, American Mathematical Society, 2004.

[3] D. A. Hejhal, *The Selberg Trace formula for* PSL(2,R), Vol. I, Lecture Notes in Mathematics 548, Springer Verlag, 1976.

[4] D. A. Hejhal, *The Selberg Trace formula for* PSL(2,R), Vol. II, Lecture Notes in Mathematics 1001, Springer Verlag, 1983.

[5] J. Fischer, *An Approach to the Selberg Trace Formula via the Selberg Zeta Function*, Lecture Notes in Mathematics 1253, Springer Verlag, 1987.

Grading

	Criterion	Points	Cut-off points
1.	Written assignment	20	11
2.	Project	40	22
3	Final exam	40	22
	Total	100	55